

BLACK HOLE THERMODYNAMICS TODAY^a

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A brief survey of the major themes and developments of black hole thermodynamics in the 1990's is given, followed by summaries of the talks on this subject at MG8 together with a bit of commentary, and closing with a look towards the future.

1 Black hole thermodynamics in the 1990's

The subject of black hole thermodynamics was born some twenty-five years ago and it is still a source of much hope and mystification. Except for a lull during the 1980's, there has been vigorous research activity pursuing the promise held out by this confluence of gravitation, quantum field theory, and thermodynamics. After initial incredulity at Bekenstein's suggestion that the area of an event horizon really is a measure of the entropy of the black hole, we have been led by a myriad of interconnecting results to the firm conclusion that it is indeed so.

It is the beauty of thermodynamics that no account of the underlying microscopic details is required in order to deduce fundamental relations between macroscopic quantities for systems in (quasi-)equilibrium. This is why we feel we now have a foot in the quantum gravity door. However, to get another foot inside, we need to pursue the microscopic description. This pursuit seems to amount to basically two questions: how does the gravitational field microscopically react to the quantum process of Hawking radiation, and what are the states which are counted by the black hole entropy?

The resurgence of interest in black hole thermodynamics (BHT) in the 1990's owes a lot to two waves of influence from string theory. The first wave came with the two-dimensional string-inspired models of "dilaton gravity". These models, coupled to matter, are two-dimensional quantum field theories which possess black hole solutions that exhibit Hawking radiation and which hold out the promise of being fully understandable if not exactly soluble.¹ The second wave came with the introduction of D-branes as nonperturbative stringy objects on which open strings can terminate. D-branes made it possible to interpolate between special highly supersymmetric black holes and perturbative string physics. This correspondence has provided an account for the black hole entropy on the stringy side in terms of an enumeration of string states, and has provided a description of black hole radiation in terms of unitary emission of closed strings from D-branes when pairs of open strings annihilate.² The string results are in quantitatively precise agreement with computations of entropy and Hawking flux based on the utterly different framework of classical gravity and curved space quantum field theory. This astonishing

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agreement may be simply a consequence of very powerful constraints imposed by the high degree of supersymmetry, however some detailed correspondence has also been found to extend beyond the supersymmetric configurations.³

Besides the string infusion, several other developments have stimulated much research on BHT in the 1990's. I hope the reader will forgive me for just mentioning a few that have been particularly influential. One is the extension of black hole thermodynamics to allow for the higher curvature couplings that arise in any low energy effective action (and in particular in the stringy one), both at the classical and quantum levels.⁴ A related development was the important realization that the divergences in the entropy of the thermal bath of acceleration radiation (or, (sometimes) equivalently, in the entanglement entropy of quantum field fluctuations across the event horizon) are precisely absorbed in the renormalization of Newton's constant and the other parameters in the effective action that appear in the expression for the black hole entropy.⁵ This insight supports the recurring suspicion that black hole entropy is associated with vacuum fluctuations of quantum fields.

Another development of the 90's was sparked by the discovery that black holes exist in three dimensional gravity with a negative cosmological constant, a theory with no local degrees of freedom at all which had already been extensively studied as a model for quantum gravity.⁶ The paradox of how these "topological black holes" can have a huge entropy seems sure to lead to important insights about quantum gravity even in four dimensions. A partial understanding of the state counting of "would-be" gauge degrees of freedom associated with the horizon has been achieved already.⁷

More recently, an approach to black hole entropy is emerging from the other major program in quantum gravity, loop quantization. The area operator is perhaps the simplest and most natural operator in this approach and is diagonal on the spin network states. The area of a surface is determined by the spins on the lines that puncture the surface, and the number of ways to obtain the same area is the exponential of the area times an unknown constant. The famous factor $1/4\hbar G$ is not yet accounted for in this approach, since doing so would require somehow connecting the microscopic quantum gravity theory to the low energy effective Newton constant, which is something that has not yet been achieved. To make this connection would appear to require dealing with the dynamics of the theory, an aspect that is not yet understood. Perhaps the black hole problem will be of some help in making that connection.

It is curious how utterly different the accounts of black hole entropy seem to be according to the different lines of approach being taken. This may be simply because one is only testing self-consistency of theories, and nothing is being learned about Nature. However, it could be that one of these points of view will eventually make itself known as the right one, perhaps the only one that really works, thus guiding us towards the right theory of quantum gravity. More likely, however, seems the possibility that the commingling of these ideas will lead to a synthesis that transcends any one of them.

2 Black hole thermodynamics at MG8

Four plenary talks were devoted to black hole thermodynamics at MG8. Bekenstein spoke on the hypothesis that the black hole mass has a discrete spectrum yielding a uniformly spaced area spectrum, which leads to a proportionality between area and entropy and to a distortion of the semiclassical Hawking spectrum. Parentani argued that when the gravitational field is allowed to react, transition amplitudes for *matter* are given by differences of the total *gravitational* action in the WKB approximation. He illustrated this using both (homogeneous) quantum cosmology and Schwinger pair creation. In the latter case, the gravitational action change is one-fourth the change in area of the acceleration horizon, which demonstrates that not just black hole horizons but also acceleration horizons possess a thermodynamic entropy which plays a role in dynamics, something which was first noticed in the context of black hole pair creation. Maldacena spoke on black holes and D-branes, and Teitelboim spoke on 2+1 dimensional black holes.

With a couple of exceptions, the talks at the BHT parallel sessions were not closely related to the subjects of any of these four plenary talks, although most were related somehow to one or more of the themes sketched above. This serves to emphasize that there are quite a few active lines of approach to BHT.

Fifteen talks were given at the BHT sessions, which spanned a total time of about six hours. All attendees who submitted abstracts spoke. (Abstracts were also submitted by five people who ultimately were unable to attend the meeting.) I will briefly describe these talks here, grouping them into the two categories of *Entropy* and *Hawking radiation and back-reaction*. Where the titles of the contributed papers are different from those of the corresponding talks at MG8, I use here the paper titles so as to facilitate cross referencing in these proceedings. In the case of multiple authors, the one who spoke is indicated with an asterisk. References are given below only where there is no associated contribution to these proceedings.

2.1 Entropy

Six of the talks dealt with microscopic accounts of the black hole entropy, while two focused on macroscopic aspects of the concept of black hole entropy extended beyond the usual black hole setting. In the order discussed here these were:

Induced entropy of a black hole in Sakharov's induced gravity

V.P. Frolov

Quantum entropy of charged rotating black holes

*R.B. Mann** and *S. Solodukhin*

Black hole entropy and entanglement thermodynamics

*H. Kodama**, *S. Mukohyama*, and *M. Seriu*

Rindler space entropy, *J.R.A. Salazar** and *J.M.T. Sarmiento*

Black hole entropy from loop quantum gravity, *C. Rovelli*

The black hole entropy: a spacetime foam approach, *F. Scardigli*

Black holes of constant curvature, *M. Banados*

The entropy of instantons with NUT charge
*S.W. Hawking and C.J. Hunter**

The first four talks listed above were concerned with the idea of attributing the black hole entropy to the entropy of quantum fields associated with horizons. This entropy, which goes by various names, can be viewed either as the entanglement entropy across the horizon, or as the entropy of the thermal bath of acceleration radiation (i.e. Rindler or Boulware quanta) outside the horizon. For certain types of matter fields (minimally coupled scalars and spinor fields) this definition of the matter field entropy is adequate. However, for nonminimally coupled scalars or vector fields, one has extra contributions to the entropy of the gravitating canonical ensemble which arise from the variation of the free energy with respect to the temperature dependence of the mass of the background black hole.⁹ It seems to me quite natural to include this variation in the definition of the entropy of the acceleration radiation, though it is not so easy to see how to modify the notion of entanglement entropy—which is an information-theoretic entropy—to accommodate the extra contributions for nonminimally coupled scalars and vectors. One idea is to presume that fields of this type are composites, which resolve at very short distances into fundamental scalars or spinors.¹⁰ Another idea is that if the notion of entanglement is somehow extended to take into account the quantum fluctuations of the horizon, then the extra contributions to the entropy might be incorporated into the entanglement entropy. It should be emphasized, however, that the contribution to the entropy in question can even be negative, so the generalization of the notion of entanglement entropy could not be just the information-theoretic entropy of a single density matrix.

Frolov discussed a model “induced gravity” field theory in which the bare inverse Newton constant G_B^{-1} vanishes and the divergent contributions to the renormalized coupling G_R^{-1} cancel at one loop so that G_R^{-1} is finite.¹¹ (To simplify matters the higher derivative terms were ignored in this treatment but they could be handled similarly.) In this theory the black hole entropy arises entirely from quantum matter field fluctuations, so an interpretation of the entropy in terms of a counting of quantum field states should be possible. A straightforward interpretation in terms of the entropy S^{SM} of the acceleration radiation outside the black hole is frustrated by the contributions to the entropy from the nonminimal couplings $\xi\phi^2R$ (cf. the discussion in the previous paragraph), which are required to obtain a finite result for G_R^{-1} . Frolov argued that the discrepancy occurs because S^{SM} counts the number of states at a fixed value of the *hamiltonian* whereas the black hole entropy counts the number of states with a given *energy*. The hamiltonian differs from the energy in this theory by a boundary term proportional to the non-minimal couplings ξ , which accounts for the difference $S^{BH} - S^{SM}$. I might emphasize that Frolov’s approach here is to compute a *microcanonical* entropy via a state counting. If one instead computes the entropy of the *canonical* ensemble, I believe—as Frolov himself first argued—that the extra terms arising from the non-minimal couplings are produced as a result of the variation of the black hole mass as a function of the temperature of the ensemble. The relation between the microcanonical and canonical viewpoints in this context deserves to be clarified.

Mann spoke on work which extends to the case of rotating black holes prior results which established that, for static black holes, the entropy of acceleration radiation of minimally coupled massless scalar fields is precisely accounted for by the one loop renormalization of the gravitational action, including the divergent parts. An interesting detail he mentioned is that there are divergences that cannot be absorbed into curvature counter-terms in the action but which “cancel non-trivially” on the Kerr-Newman backgrounds. I would guess this cancellation can be understood as a general consequence of the fact that the background field equations are satisfied.

Kodama’s talk explored the proposal that the black hole entropy is nothing but the entanglement entropy of vacuum fluctuations across the horizon. Introducing also an “entanglement energy”, which (according to one of two definitions considered) is the difference of the energy in the original state and in the reduced state with the correlations across the horizon (or other dividing surface) taken out, he defined an entanglement temperature $T_{\text{ent}} = dS_{\text{ent}}/dE_{\text{ent}}$. The calculations are done with a short distance cutoff a in place. This temperature T_{ent} diverges as $1/a$ in flat spacetime. However, in a black hole spacetime, if the entanglement entropy is first redshifted from to infinity from a proper distance a from the horizon, then the temperature comes out to be of order the Hawking temperature. This result is closely related to the frameworks discussed by Frolov and Mann. It seems particularly close in spirit to Frolov’s identification of the entropy as arising from counting the states with the same quantum field energy outside the horizon, although the non-minimal coupling plays no role in Kodama’s work. This is a good place to mention also Salazar’s talk, in which the method of obtaining a reduced density matrix for the quantum fields in Rindler spacetime was discussed.

Rovelli presented a computation of the black hole entropy in loop quantum gravity along the lines mentioned in section 1 above.¹² The computation itself is straightforward, involving the number of ways to obtain a given area with spin networks puncturing a surface. Most of Rovelli’s talk concerned rather the conceptual underpinning of this calculation, addressing the question why should the area be held fixed in the state counting. He argued that only the shape of the horizon matters since the inside is unobservable, and that fixing the area corresponds to fixing the energy, which defines the microcanonical ensemble.

Scardigli postulated a model for the states of a black hole which assigns the entropy to the degeneracy of configurations of two-state “topological cells” of Planck area which form the surface of the horizon. He supposed that each cell can be found in two different states, in one of which it carries no energy and in the other its energy is the Planck mass. Using these ideas he computed the entropy at fixed temperature for this system, which for large black holes comes out proportional to the horizon area.

Two of the talks focused on macroscopic aspects of the concept of black hole entropy extended beyond the usual black hole setting. Bañados presented a family of constant curvature black holes obtained by quotients of anti-de Sitter space in any dimension. This construction generalizes that of the 2+1 dimensional black hole discussed in Teitelboim’s plenary talk. In the five dimensional case the energy and angular momentum of these black holes can be defined as charges of a Chern-

Simons supergravity theory. Variations of these charges satisfy a first law with an entropy that is *not* proportional to the horizon area. I did not catch any explanation of this surprising feature in the talk.

A departure from the entropy-area relation also occurred in Hunter’s talk. He spoke on the idea of attributing entropy to stationary gravitational fields which are not black holes but which do have zeroes of the Killing field, in particular four dimensional solutions to the Einstein equations with NUT charge or magnetic mass. Although these solutions can be assigned an entropy it is *not* related to the (vanishing) area of the fixed point set.

2.2 *Hawking radiation and back-reaction*

Seven of the talks dealt with aspects of Hawking radiation and the back reaction. In the order discussed here these were:

Euclidean instantons and Hawking radiation, *S. Massar** and *R. Parentani*

Covariant path integrals and black holes, *F. Vendrell** and *M.E. Ortiz*

Loop corrections for 2D Hawking radiation, *A. Miković** and *V. Radovanović*

The ‘ups’ and ‘downs’ of a spinning black hole
*C.M. Chambers**, *W.A. Hiscock*, and *B.E. Taylor*

Constraints on the geometries of black holes in classical and semiclassical gravity, *P.R. Anderson** and *C.D. Mull*

Semiclassical decay of near-extremal black holes, *T. Jacobson*

Thermodynamics of nonsingular spherically symmetric black hole
I. Dymnikowa

Massar described a quantum gravity calculation in which the rate of Hawking radiation of charged shells by a charged black hole is obtained from the difference of WKB actions of corresponding Euclidean instantons with and without the charged shell, including the gravitational back-reaction. This difference is equal to one fourth the difference in horizon areas, in agreement with Hawking’s result. Only the *difference* of actions enters the rate, because one is computing a dynamical transition amplitude, so no “regularization” of infinite actions is required. Massar argued that this relation between pair creation rate and horizon area change—which was also discussed in Parentani’s plenary talk—applies quite generally to all sorts of pair creation processes.

Vendrell described a new mathematical approach to obtaining the thermal propagator for a particle in a black hole spacetime. In this approach, only the exterior portion of the spacetime is included in the configuration space, however the tortoise coordinate is analytically continued to complex values. This yields a multiply connected complex configuration space which covers the Kruskal manifold an infinite number of times. The path integral for a particle on this space yields the propagator in the Unruh vacuum.

Chambers presented results of numerical calculations which revealed a counterexample to the usual expectation that a rotating black hole will spin down to

a final asymptotic nonrotating state. In particular, a lone massless scalar field radiates enough power in the $l = 0$ mode so that the asymptotic value of J/M^2 is approximately 0.555 rather than zero, whether it begins above or below this value. (In Nature, however, the presence of higher spin fields would spin down the black hole completely unless there is a very large number of hitherto undiscovered massless scalars.) I might add that, curiously, this result seems to suggest that a nonrotating black hole coupled only to massless scalars is unstable to spinning up to just this value of J/M^2 .

The subject of Miković’s talk was two dimensional black hole evaporation in the CGHS dilaton gravity model. By adopting the point of view of reduced phase space operator quantization, he is trying to take the study of this model further than has been previously possible. He presented a loop expansion of the metric expectation value, and argued that at two loops one can see, both from the Bogoliubov coefficients in the effective geometry and from the (more reliable) flux operator, a non-classical increase in the Hawking temperature at late times. At still later times the operator flux is non-thermal, drops to negative values, and then approaches zero. He sees indications that the higher loop corrections can remove the singularities associated with the lower order approximations.

Anderson reported on constraints on static spherically symmetric black holes imposed by the assumption that the curvature components in the static orthonormal frame are analytic functions of r at the horizon. For conformally coupled free massless fields the additional constraint imposed by the trace of the semiclassical back-reaction equations implies that extremal black holes do not exist for a certain range of horizon radii. The excluded range depends on the number and types of quantum fields, and can extend all the way to zero, in which case there is a minimum allowed radius for an extremal black hole.

Jacobson’s talk concerned the semiclassical description of decay of a near-extremal black hole down to an extremal state in the adiabatic approximation.¹³ The motivation was to try to reconcile the fact that near-extremal D-branes illuminated by a pure state energy flux do not radiate unlimited entropy whereas the corresponding black holes seem to do so. He argued that the semiclassical physics is very different than for non-extremal black holes, due to a “pile-up” of the partners of Hawking radiation inside the horizon. However, this difference does not appear sufficient to invalidate the semiclassical analysis of the radiated entropy. Thus no reconciliation with the D-brane analysis was achieved.

Dymnikowa’s talk addressed the idea that the back-reaction near a black hole singularity might remove the singularity, replacing it by a de Sitter phase inside. She postulated a simple form for the stress-energy tensor in regions of large curvature, motivated by an analogy with vacuum polarization in an electric field, that would achieve this sort of configuration if matched onto a Schwarzschild solution. Using this form she argued that an evaporating black hole would shrink down to a critical nonvanishing size at which its temperature would vanish. To determine the further evolution of this “extremal state” would seem to require a more complete dynamical picture.

3 Black hole thermodynamics in the future

Despite the advances of recent years, much remains to be understood about black hole thermodynamics. In the long run, the goal is nothing short of a full understanding of quantum gravity and what it means for singularities, the fate of black holes, unitarity, and the topology of spacetime. The day when this goal is met seems very far off indeed. However, there is plenty of reason for optimism in the short run. We can anticipate improved understanding of the state counting entropy of 2+1 dimensional and other topological black holes, including the 1+1 dimensional dilatonic topological case, which has so far resisted attempts.¹⁴ A much better understanding of 1+1 dilaton gravity coupled to matter seems possible, including a definitive statement about the fate of singularities and unitarity. If string theory can stay put long enough it should be possible for someone to understand why semi-classical black hole physics permits unlimited (entanglement) entropy production whereas D-brane physics does not. String theory (and its descendents) will continue to provide new insights, perhaps via the matrix theory description of neutral black holes and Hawking radiation.¹⁵ The loop quantization approach may well be successfully extended to count the entropy of rotating black holes, and perhaps the factor of 1/4 will be understood together with something of the dynamics in this formulation. Condensed matter analogies for black holes may provide observable instances of the Hawking effect and the decay of the ergoregion¹⁶, and it should be possible to understand in detail how the outgoing black hole modes are produced without an infinite density of states at the horizon in these systems¹⁷. It seems not unreasonable to expect that many of these and other puzzles will be resolved in the next several years, some of them in time for the next Marcel Grossmann meeting.

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